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2. Methods

2.1. Theory of low-frequency estimation

A ... $g_l(i,j), i,j \in \chi_l = \{0,1,\ldots,N-1\}$ ... $g(i,j), i,j \in \chi = \{0,1,\ldots,sN-1\}$ ... s. ... s ... (... , $s = 2$ ... $N \times N$ ... $2N \times 2N$). ... $g_l(i,j)$ ... $g(si+a, sj+b)$

$s \times s$

$$g_l(i,j) = \frac{1}{s^2}\sum_{a=0}^{s-1}\sum_{b=0}^{s-1} g(si+a, sj+b) \tag{1}$$

$a, b \in \{0, 1, \ldots, s-1\}$ $x-$ $y-$ $g(i,j), i,j \in \{0, 1, \ldots, sN-1\}$

$$G(k_i,k_j) = \mathcal{F}\ g(i,j), \quad k_i,k_j \in \left\{-\frac{sN}{2}, -\frac{sN}{2}+1, \ldots, \frac{sN}{2}-1\right\} \tag{2}$$

$\mathcal{F}$ $g(i,j)$ $k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\}$ ( )

$$\begin{aligned} G(k_i,k_j) &= \mathcal{F}\ g(i,j), \quad k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\} \\ &= \sum_{i=0}^{N-1} e^{\frac{-2\pi i k_i\sqrt{-1}}{N}} \sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i\sqrt{-1}}{sN}} \\ &\quad \times \left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g(si+a, sj+b)\, e^{\frac{-2\pi b k_j\sqrt{-1}}{sN}}\right] e^{\frac{-2\pi j k_j\sqrt{-1}}{N}}\right] \end{aligned} \tag{3}$$

$g(si+a, sj+b)$ $a, b \in \{0, 1, \ldots, s-1\}$ $i,j \in \{0, 1, \ldots, N-1\}$ $\frac{1}{s^2}\sum_{a=0}^{s-1}\sum_{b=0}^{s-1} g(si+a, sj+b)$ A (1), $g_l(i,j), i,j \in \{0, 1, \ldots, N-1\}$. $g_l(i,j)$

$$g(si+a, sj+b) \approx g_l(i,j), \quad \forall a,b = 0,1,\ldots,s-1 \quad i,j = 0,1,\ldots,N-1 \tag{4}$$

$G(k_i,k_j)$ $k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\}$ ($N \times N$), (4) $g(si+a, sj+b)$ $g_l(i,j)$ (3). $g(i,j)$ ( )

$$\begin{aligned} G(k_i,k_j) &\approx \sum_{i=0}^{N-1} e^{\frac{-2\pi i k_i\sqrt{-1}}{N}} \sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i\sqrt{-1}}{sN}} \left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g_l(i,j) e^{\frac{-2\pi b k_j\sqrt{-1}}{sN}}\right] e^{\frac{-2\pi j k_j\sqrt{-1}}{N}}\right] \\ &= \left(\sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i\sqrt{-1}}{sN}}\right)\left(\sum_{b=0}^{s-1} e^{\frac{-2\pi b k_j\sqrt{-1}}{sN}}\right) G_l(k_i,k_j), \\ k_i,k_j &\in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\} \end{aligned} \tag{5}$$

$$G_l(k_i,k_j) = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} e^{\frac{-2\pi(i k_i + j k_j)\sqrt{-1}}{N}} g_l(i,j), \quad k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\} \tag{6}$$

(5) (4). ( ) ( ) (5) $k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\}$ $g(i,j)$ $G_l(k_i,k_j)$

$$\begin{aligned} W_s(k_i,k_j) &= \left(\sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i\sqrt{-1}}{sN}}\right)\left(\sum_{b=0}^{s-1} e^{\frac{-2\pi b k_j\sqrt{-1}}{sN}}\right), \\ k_i,k_j &\in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, -\frac{N}{2}-1\right\} \end{aligned} \tag{7}$$

$k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\}$

$$G(k_i,k_j) \approx W_s(k_i,k_j)\, G_l(k_i,k_j), \quad k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\} \tag{8}$$

$W_s(k_i,k_j)$. 1. $k_i,k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\}$ (8)

2.2. Reconstruction method

$g(i,j), i,j \in \chi$ $g_l(i,j), i,j \in \{0, 1, \ldots, N-1\}$. $G(k_i,k_j)$, $k_i,k_j \in \Omega = \left\{-\frac{sN}{2}, -\frac{sN}{2}+1, \ldots, \frac{sN}{2}-1\right\}$. A (8), ( ) $\Omega_l = \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\}$ $W_s(k_i,k_j)G_l(k_i,k_j)$, $k_i,k_j \notin \Omega_l$. $\Omega - \Omega_l$ $g(i,j), i,j \in \chi$

$$G(k_i,k_j) \quad (k_i,k_j) \approx \begin{cases} G_l(k_i,k_j)\, W_s(k_i,k_j), & k_i,k_j \in \Omega_l \\ 0, & k_i,k_j \in \Omega - \Omega_l \end{cases} \tag{9}$$

$$(k_i,k_j) = \begin{cases} 1, & k_i,k_j \in \Omega_l \\ 0, & k_i,k_j \in \Omega - \Omega_l \end{cases} \tag{10}$$

$g(i,j), i,j \in \chi$

$$g(i,j) \approx \mathcal{F}^{-1}\left[G(k_i,k_j) \quad (k_i,k_j)\right], \quad i,j \in \chi \tag{11}$$

$\mathcal{F}^{-1}$

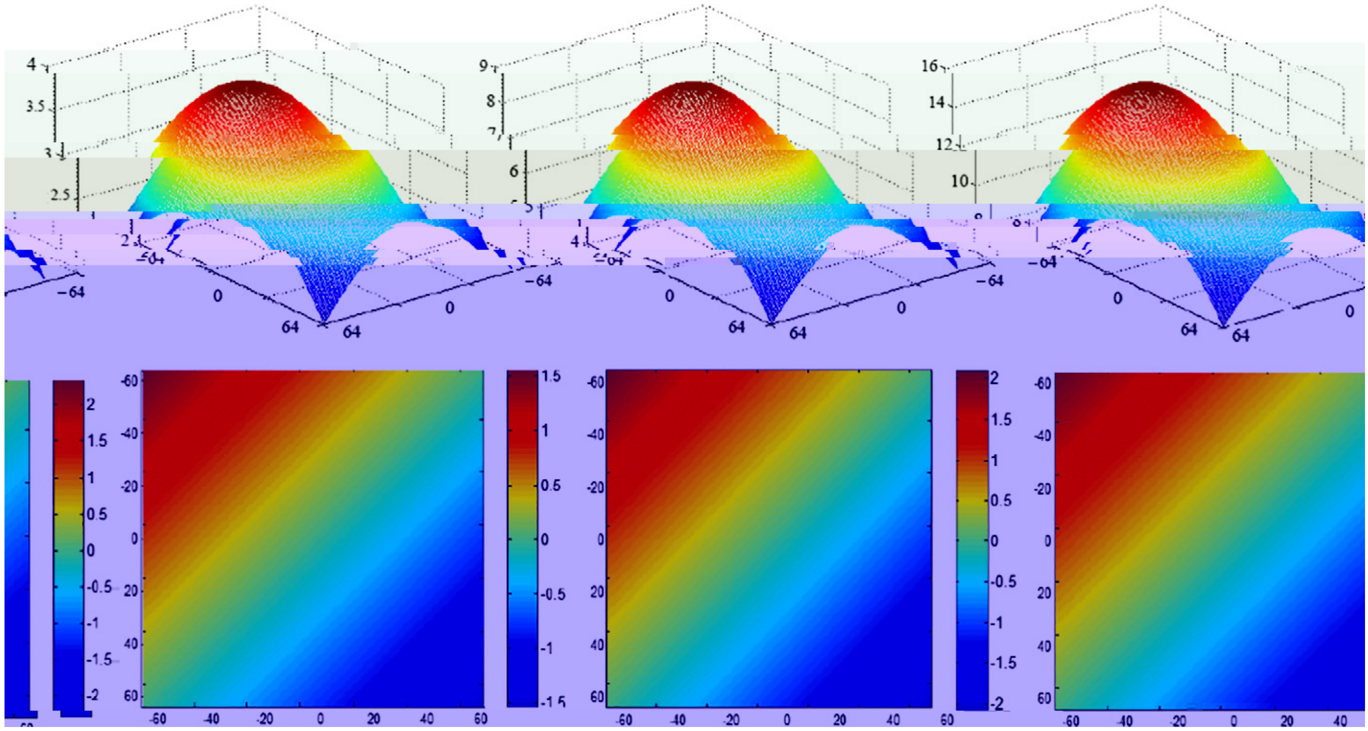


Fig. 1. ( ) ( ) $W_s(k_i, k_j)$ $s = 2, 3, 4$ ( ).

(11)

$G_l(k_i, k_j)W_s(k_i, k_j)$

(9)

( ).

(9)

s ((7)).

( )

((8)),

.2 $g(i,j)$

$g_l(i,j)$

$W_s(k_i, k_j)G_l(k_i, k_j)$ $g(i,j)$

3. Experiments and results

256×256, 1

15.

( ) $1/255, 2/255, \ldots, 15/255$,

1 $= 1/255$, 2

$= 2/255$,

( ).

1 5 33.[1]

$1.0 \times 1.0 \times 1.0$

256 256×256.

0.9375×0.9375, 1.3 , 124

256×256).

31, 14, 20, 22,

$W_s(k_i, k_j) = 1, k_i, k_j \in \Omega_l$ .2.

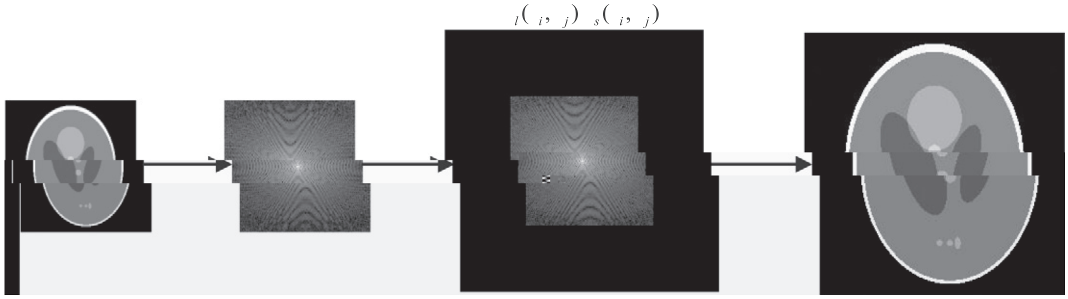
14,

4

20, 22,

$g(i,j), i,j \in \chi = \{0, 1, \ldots, sN-1\}$

[1] :// /541



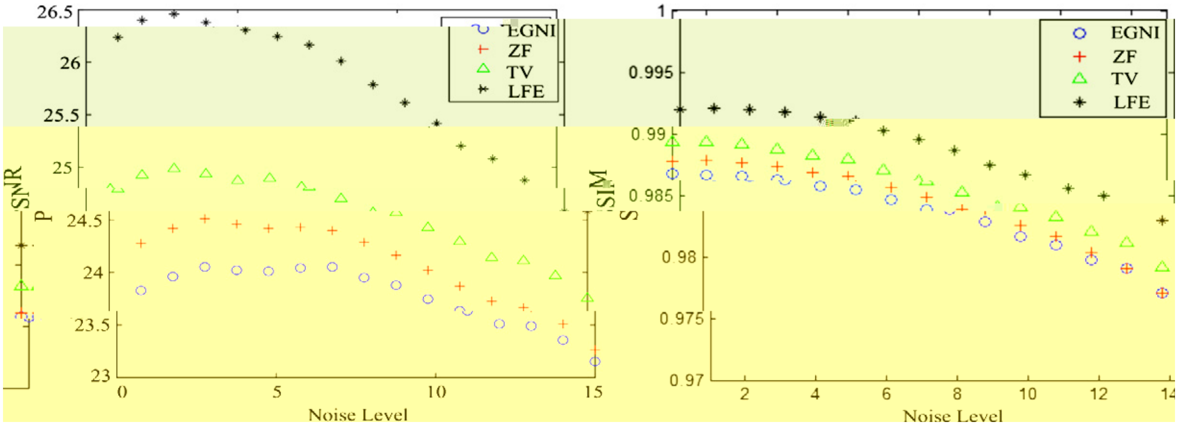


Fig. 3.

3.3. Reconstruction results on real brain MR images

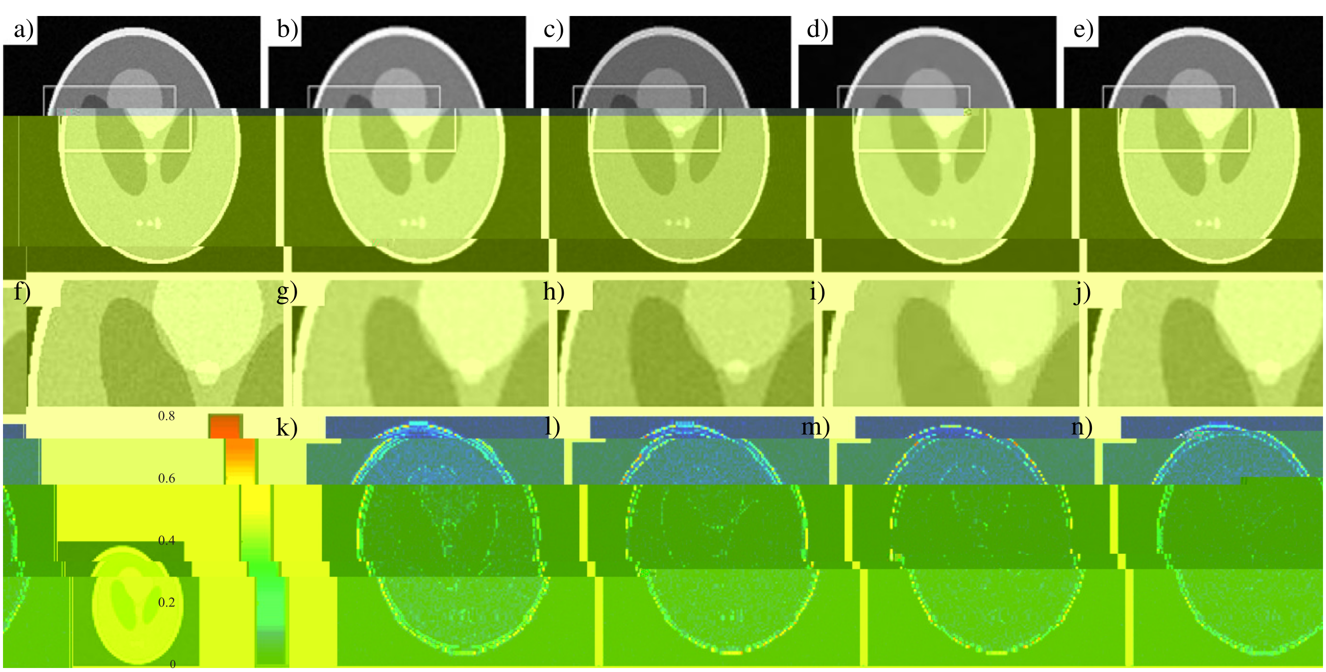
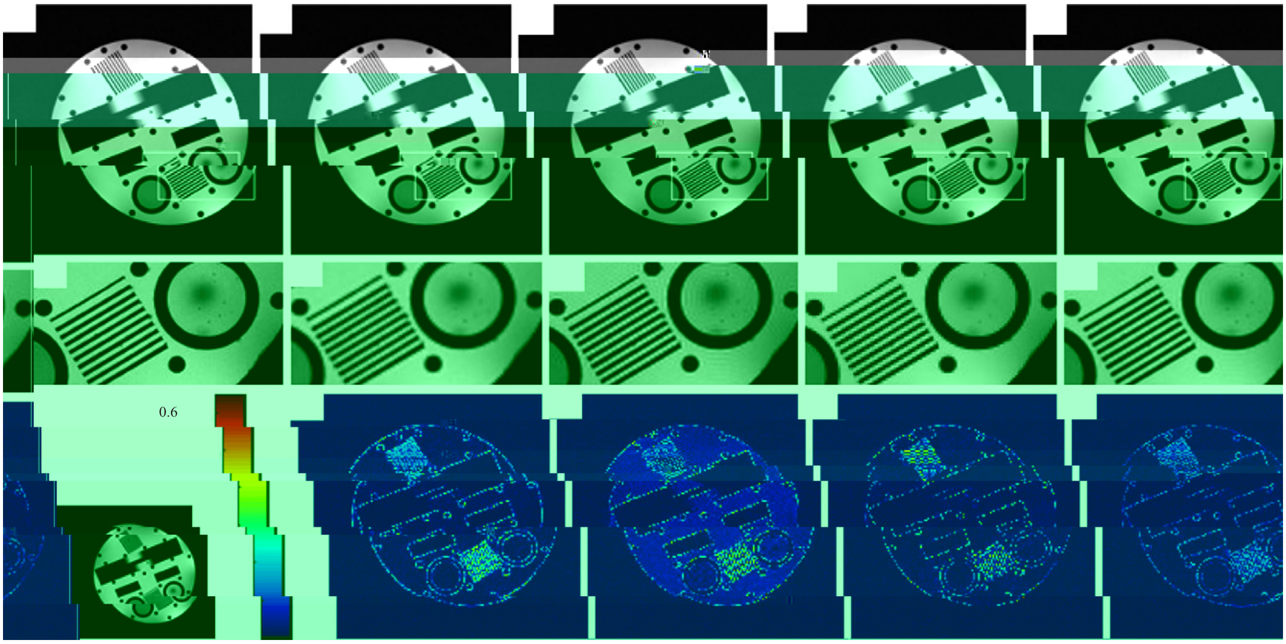


Fig. 4.


0.6

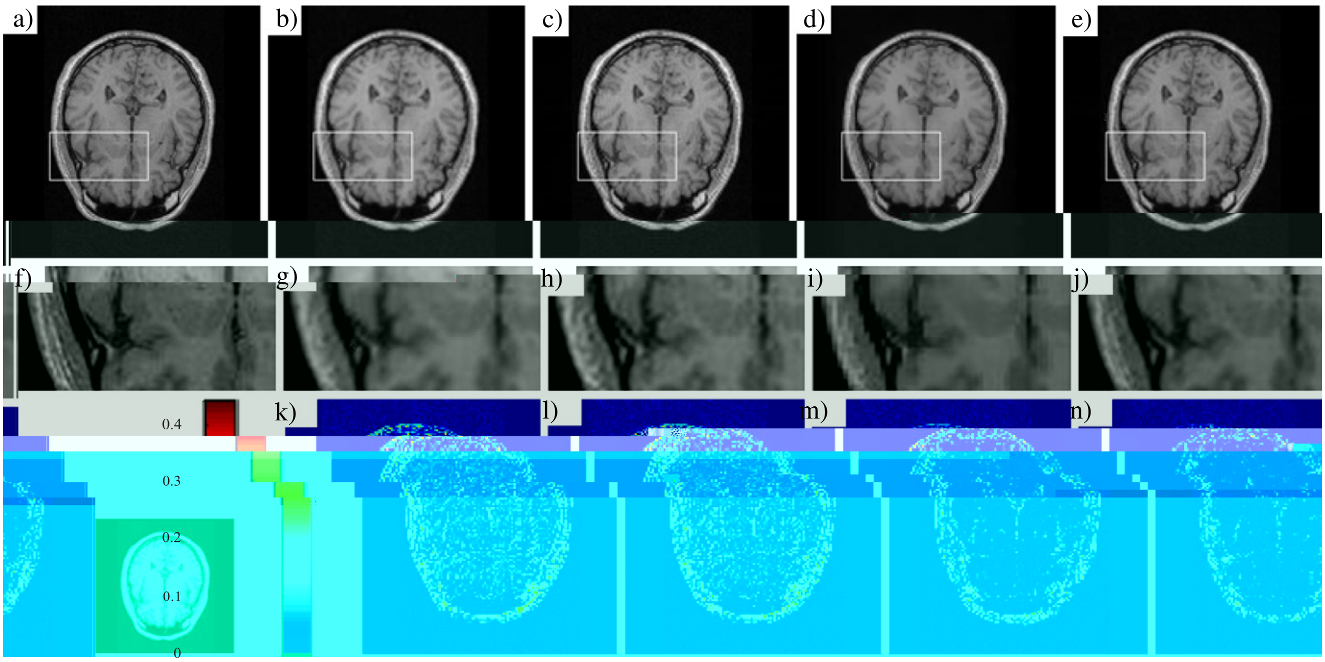


Fig. 7. 100

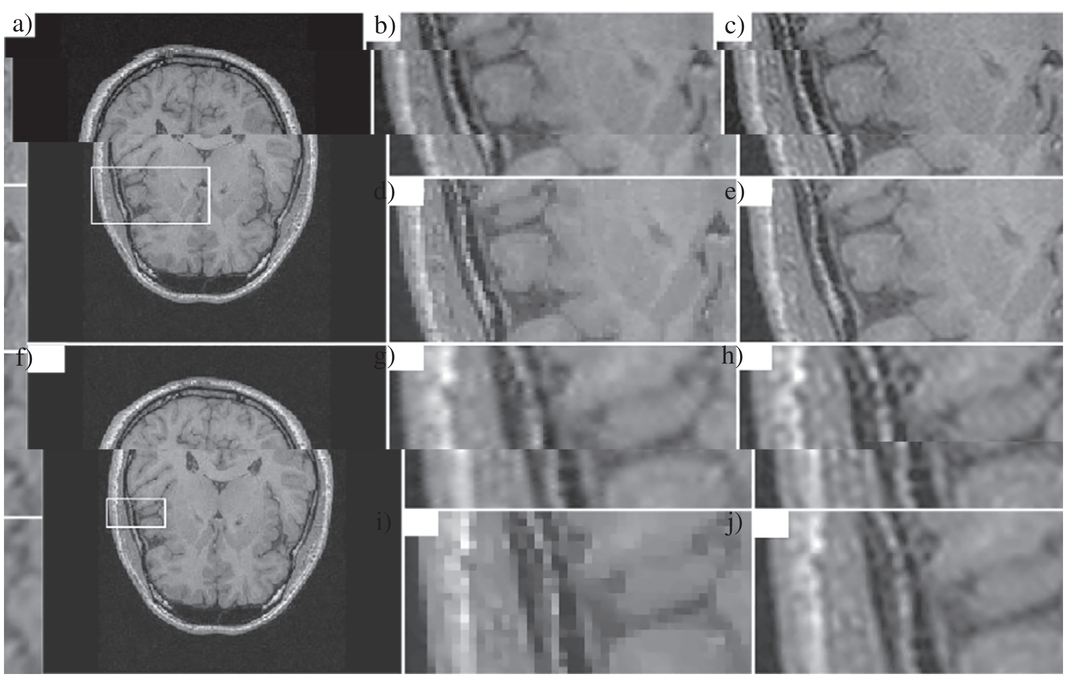


Fig. 8. 204 × 176. 2, 4.

Table 2

	1	2	3	4	5
[illegible]	39	160	39	159	789
[illegible]	0.003	0.016	0.004	0.015	0.078
[illegible]	0.86	3.5	0.82	3.5	8.90
[illegible]	0.003	0.016	0.005	0.018	0.083

2 A 5-4460 3.2 4

Table 3

1	29.0602	29.3056	30.6283	30.8036	0.9816	0.9817	0.9907	0.9913
2	28.7750	28.9821	29.9099	30.0607	0.9892	0.9880	0.9920	0.9925
3	29.5882	30.0161	31.2284	31.4380	0.9797	0.9815	0.9895	0.9902
4	29.5255	30.0043	31.3300	31.6244	0.9783	0.9815	0.9899	0.9908
5	28.9796	29.1264	30.4487	30.6215	0.9806	0.9782	0.9891	0.9899
6	25.6576	26.3935	29.1219	29.5618	0.9598	0.9685	0.9811	0.9925
7	23.2625	23.6571	26.6679	27.2574	0.8953	0.9132	0.9712	0.9891
8	25.9609	26.7609	28.3118	29.1991	0.9775	0.9844	0.9820	0.9930

4. Discussion and conclusions

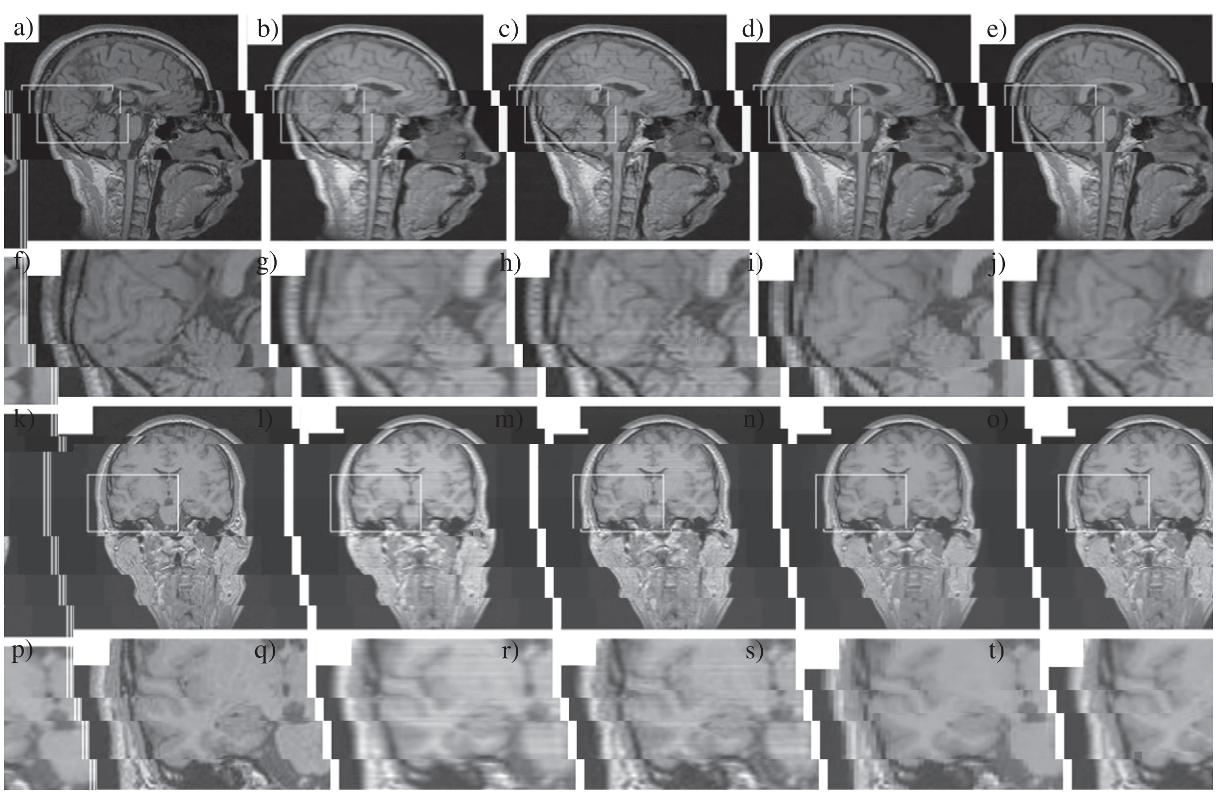


Fig. 9.

. A ,

(3) .

Acknowledgments

A

(. 204 22, 2015), (. 61271320) (2014 29), -A A A 2013340. .

Appendix

1. $g(i,j), i,j \in \{0,1,\dots,sN-1\}$ $k_i,k_j \in \left\{-\frac{N}{2},-\frac{N}{2}+1,\dots,\frac{N}{2}-1\right\}$:

$$
\begin{aligned}
G(k_i,k_j) &= \mathcal{F}\ g(i,j), \quad k_i,k_j \in \left\{-\frac{N}{2},-\frac{N}{2}+1,\dots,\frac{N}{2}-1\right\} \\
&= \sum_{i=0}^{sN-1}\sum_{j=0}^{sN-1} g(i,j)e^{\frac{-2\pi\left(ik_i+jk_j\right)\sqrt{-1}}{sN}} \\
&= \sum_{i=0}^{sN-1} e^{\frac{-2\pi ik_i\sqrt{-1}}{sN}}\left[\sum_{j=0}^{N-1}\left[g(i,sj)e^{\frac{-2\pi sjk_j\sqrt{-1}}{sN}} + \dots + g(i,sj+s-1)\,e^{\frac{-2\pi(sj+s-1)k_j\sqrt{-1}}{sN}}\right]\right] \\
&= \sum_{i=0}^{sN-1} e^{\frac{-2\pi ik_i\sqrt{-1}}{sN}}\left[\sum_{j=0}^{N-1}\left[g(i,sj) + \dots + g(i,sj+s-1)\,e^{\frac{-2\pi(s-1)k_j\sqrt{-1}}{sN}}\right]e^{\frac{-2\pi jk_j\sqrt{-1}}{N}}\right] \\
&= \sum_{i=0}^{sN-1} e^{\frac{-2\pi ik_i\sqrt{-1}}{sN}}\left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g(i,sj+b)e^{\frac{-2\pi bk_j\sqrt{-1}}{sN}}\right]e^{\frac{-2\pi jk_j\sqrt{-1}}{N}}\right] \\
&= \sum_{i=0}^{N-1} e^{\frac{-2\pi sik_i\sqrt{-1}}{sN}}\left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g(i,sj+b)e^{\frac{-2\pi bk_j\sqrt{-1}}{sN}}\right]e^{\frac{-2\pi jk_j\sqrt{-1}}{N}}\right] \\
&\quad + \sum_{i=0}^{N-1} e^{\frac{-2\pi(si+1)k_i\sqrt{-1}}{sN}}\left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g(si+1,sj+b)e^{\frac{-2\pi bk_j\sqrt{-1}}{sN}}\right]e^{\frac{-2\pi jk_j\sqrt{-1}}{N}}\right] \\
&\quad \dots \\
&\quad + \sum_{i=0}^{N-1} e^{\frac{-2\pi(si+s-1)k_i\sqrt{-1}}{sN}}\left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g\left(si+s-1,\ s\right.\right.\right.
\end{aligned}
$$

2. $G(k_i, k_j)$ $k_i, k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right\}$ ($N \times N$), $g(si+a, sj+b)$ $a, b \in \{0, 1, \ldots, s-1\}$ $i, j \in \{0, 1, \ldots, N-1\}$ $g_l(i,j), i,j \in \{0, 1, \ldots, N-1\}$. $g(i,j)$:

$$
\begin{aligned}
G\left(k_i, k_j\right) &= \sum_{i=0}^{N-1} e^{\frac{-2\pi i k_i \sqrt{-1}}{N}} \sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i \sqrt{-1}}{sN}} \left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g(si+a, sj+b) e^{\frac{-2\pi b k_j \sqrt{-1}}{sN}}\right] e^{\frac{-2\pi j k_j \sqrt{-1}}{N}}\right] \\
&\approx \sum_{i=0}^{N-1} e^{\frac{-2\pi i k_i \sqrt{-1}}{N}} \sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i \sqrt{-1}}{sN}} \left[\sum_{j=0}^{N-1}\left[\sum_{b=0}^{s-1} g_l(i,j) e^{\frac{-2\pi b k_j \sqrt{-1}}{sN}}\right] e^{\frac{-2\pi j k_j \sqrt{-1}}{N}}\right] \\
&= \sum_{i=0}^{N-1} e^{\frac{-2\pi i k_i \sqrt{-1}}{N}} \sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i \sqrt{-1}}{sN}} \left[\sum_{j=0}^{N-1}\left[g_l(i,j) \sum_{b=0}^{s-1} e^{\frac{-2\pi b k_j \sqrt{-1}}{sN}}\right] e^{\frac{-2\pi j k_j \sqrt{-1}}{N}}\right] \\
&= \left(\sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i \sqrt{-1}}{sN}}\right)\left(\sum_{b=0}^{s-1} e^{\frac{-2\pi b k_j \sqrt{-1}}{sN}}\right) \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} e^{\frac{-2\pi (i k_i + j k_j)\sqrt{-1}}{N}} g_l(i,j) \\
&= \left(\sum_{a=0}^{s-1} e^{\frac{-2\pi a k_i \sqrt{-1}}{sN}}\right)\left(\sum_{b=0}^{s-1} e^{\frac{-2\pi b k_j \sqrt{-1}}{sN}}\right) G_l\left(k_i, k_j\right),\ k_i, k_j \in \left\{-\frac{N}{2}, -\frac{N}{2}+1, \ldots, \frac{N}{2}-1\right.
\end{aligned}
$$